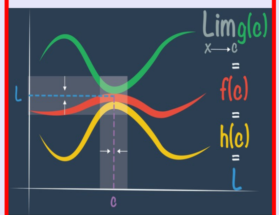
# Calculus I

# Lecture 9




Feb 19-8:47 AM

Class Quiz 9

Box Your Answers

Given $f(x) = \begin{cases} x^2 & \text{if } x<0 \\ \sqrt{x}+2 & \text{if } x \geq 0 \end{cases}$

1) Find $\lim_{x \to 0^-} f(x) = 0^2 = \boxed{0}$ ($x<0$)

2) Find $\lim_{x \to 0^+} f(x) = \sqrt{0}+2 = \boxed{2}$ ($x>0$)

3) Find $\lim_{x \to 0} f(x)$ $\boxed{\text{DNE}}$

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

4) Find $f(0) = \sqrt{0}+2 = \boxed{2}$

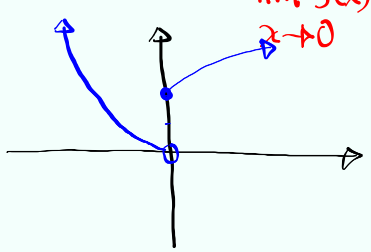
5) Is $f(x)$ continuous at $x=0$? NO

$\lim_{x \to 0} f(x) = f(0)$

DNE = 2

false

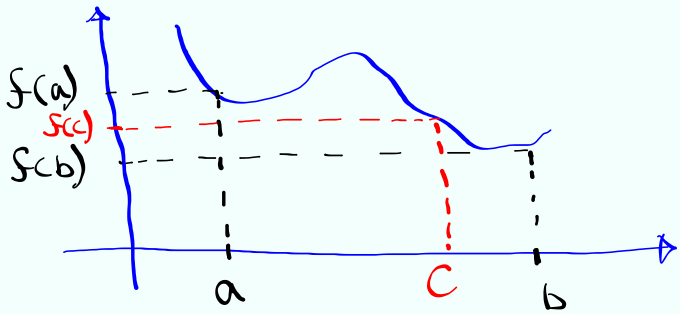
$\lim_{x \to 0} f(x)$ DNE.



Mar 12-8:17 AM

Intermediate Value Theorem (IVT)

Suppose $f(x)$ is a continuous function over $[a,b]$, then there is at least a number $c$ in $(a,b)$ such that $f(c)$ is between $f(a)$ and $f(b)$.




Mar 12-9:07 AM

Consider $f(x) = x^2$ on $[1,3]$

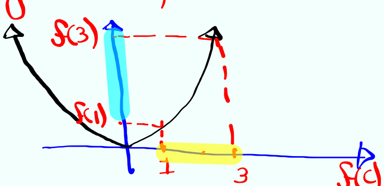
there is a number $c$ in $(1,3)$ such that $f(c)$ is between $f(1)$ and $f(3)$.

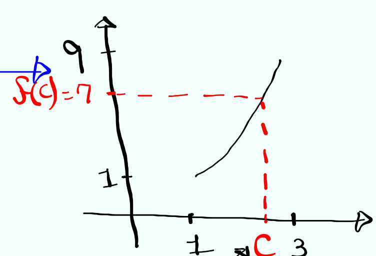
$f(x)$ is continuous $(-\infty,\infty)$

by I.V.T., there is at least one number $c$

$f(1) = 1^2 = 1$

$f(3) = 3^2 = 9$




$c^2 = 7$

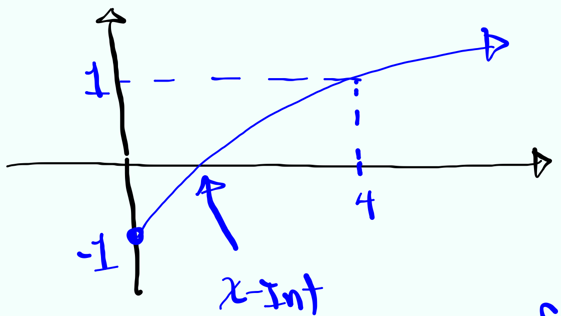
$c = \pm\sqrt{7} \to c = \sqrt{7}$

$\approx 2.65$

Mar 12-9:12 AM

Show that $f(x)=\sqrt{x}-1$ has x-Int

between $x=0$ and $x=4$.




$f(x)=\sqrt{x}-1$ is cont. on $[0,\infty)$

$f(x)=\sqrt{x}-1$ is cont. on $[0,4]$

$f(0)=\sqrt{0}-1=-1$

$f(4)=\sqrt{4}-1=2-1=1$

by I.V.T., there is at least one number c where $f(c)=0$ $\to$ $\sqrt{x}-1=0$

$\sqrt{x}=1 \to \boxed{x=1}$

Mar 12-9:18 AM

Show the equation below has a root between $\boxed{1 \text{ and } 2.}$

root: $f(c)=0$

$$4x^3+3x=6x^2+2$$

Make RHS $=0$

$4x^3+3x-6x^2-2=0$

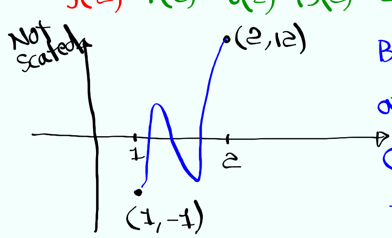
$\underbrace{4x^3-6x^2+3x-2}_{\text{Polynomial Function}}=0$

$f(x)=4x^3-6x^2+3x-2$

1) Polynomials are continuous $(-\infty,\infty)$

2) $f(1)=4(1)^3-6(1)^2+3(1)-2=-1$

$f(2)=4(2)^3-6(2)^2+3(2)-2=12$



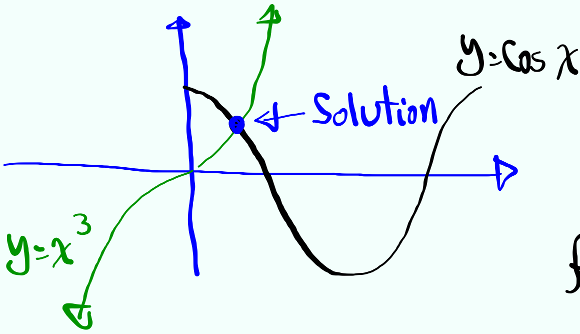

By I.V.T., there is at least a number c in $(1,2)$ such that $f(c)=0$.

Mar 12-9:23 AM

Show that $\cos x = x^3$ has a solution in interval $(0,1)$.

$\cos x = x^3$

$\underbrace{\cos x - x^3} = 0$




$f(x) = \cos x - x^3$

and is cont. $(0,1)$

$f(0) = \cos 0 - 0^3 = 1 - 0 = \boxed{1}$

$f(1) = \cos 1 - 1^3 \approx \boxed{-.5}$

By I.V.T., there is a number C in $(0,1)$ such that $f(C) = 0$

C is the Solution

Mar 12-9:29 AM

Show $f(x) = \begin{cases} x^4 \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

is continuous on $(-\infty, \infty)$

$\lim_{x \to a} f(x) = f(a)$

focus on $x=0$ since we can do all values on $(-\infty, \infty)$ except 0.

1) $f(0) = 0$

2) $\lim_{x \to 0} f(x)$

Recall from Trig.

$-1 \le \sin A \le 1$

$-1 \le \sin\frac{1}{x} \le 1$

Multiply by $x^4 \ge 0$

$x^4 \cdot (-1) \le x^4 \sin\frac{1}{x} \le x^4 \cdot 1$

$-x^4 \le x^4 \sin\frac{1}{x} \le x^4$

$\lim_{x \to 0} -x^4 = 0$, $\lim_{x \to 0} x^4 = 0$

By S.T.

$\lim_{x \to 0} x^4 \sin\frac{1}{x} = 0$

Now

$\lim_{x \to 0} x^4 \sin\frac{1}{x} = f(0)$

$\therefore$ $f(x)$ is cont. at 0.

Mar 12-9:36 AM